When they are available, then, independence assertions can help in reducing the size of the domain representation and the complexity of the inference problem. Unfortunately, clean separation of entire sets of variables by independence is quite rare. Whenever a connection, however indirect, exists between two variables, independence will fail to hold. Moreover, even independent subsets can be quite large—for example, dentistry might involve dozens of diseases and hundreds of symptoms, all of which are interrelated. To handle such problems, we need more subtle methods than the straightforward concept of independence.

12.5 Bayes' Rule and Its Use

On page 390, we defined the **product rule** (Equation (12.4)). It can actually be written in two forms:

$$P(a \wedge b) = P(a \mid b)P(b) \qquad \text{and} \qquad P(a \wedge b) = P(b \mid a)P(a)\,.$$

Equating the two right-hand sides and dividing by $P(a)$, we get

$$P(b \mid a) = \frac{P(a \mid b)P(b)}{P(a)}\,. \tag{12.12}$$

This equation is known as **Bayes' rule** (also Bayes' law or Bayes' theorem). This simple equation underlies most modern AI systems for probabilistic inference. Bayes' rule

The more general case of Bayes' rule for multivalued variables can be written in the **P** notation as follows:

$$\mathbf{P}(Y \mid X) = \frac{\mathbf{P}(X \mid Y)\mathbf{P}(Y)}{\mathbf{P}(X)}\,.$$

As before, this is to be taken as representing a set of equations, each dealing with specific values of the variables. We will also have occasion to use a more general version conditionalized on some background evidence **e**:

$$\mathbf{P}(Y \mid X, \mathbf{e}) = \frac{\mathbf{P}(X \mid Y, \mathbf{e})\mathbf{P}(Y \mid \mathbf{e})}{\mathbf{P}(X \mid \mathbf{e})}\,. \tag{12.13}$$

12.5.1 Applying Bayes' rule: The simple case

On the surface, Bayes' rule does not seem very useful. It allows us to compute the single term $P(b \mid a)$ in terms of three terms: $P(a \mid b)$, $P(b)$, and $P(a)$. That seems like two steps backwards; but Bayes' rule is useful in practice because there are many cases where we do have good probability estimates for these three numbers and need to compute the fourth. Often, we perceive as evidence the *effect* of some unknown *cause* and we would like to determine that cause. In that case, Bayes' rule becomes

$$P(\mathit{cause} \mid \mathit{effect}) = \frac{P(\mathit{effect} \mid \mathit{cause})P(\mathit{cause})}{P(\mathit{effect})}\,.$$

The conditional probability $P(\mathit{effect} \mid \mathit{cause})$ quantifies the relationship in the **causal** direction, whereas $P(\mathit{cause} \mid \mathit{effect})$ describes the **diagnostic** direction. In a task such as medical diagnosis, we often have conditional probabilities on causal relationships. The doctor knows $P(\mathit{symptoms} \mid \mathit{disease})$ and wants to derive a diagnosis, $P(\mathit{disease} \mid \mathit{symptoms})$. Causal Diagnostic

For example, a doctor knows that the disease meningitis causes a patient to have a stiff neck, say, 70% of the time. The doctor also knows some unconditional facts: the prior