Rational decisions

Chapter 16

Outline

◇ Rational preferences

◇ Utilities

◇ Money

◇ Multiattribute utilities

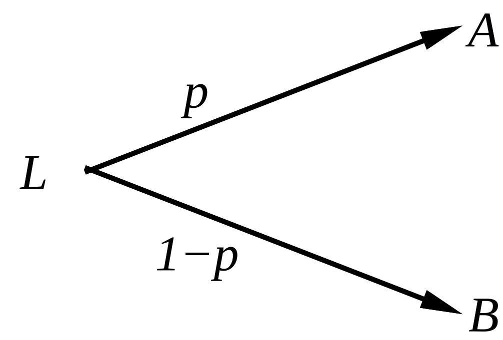
◇ Decision networks

◇ Value of information

Preferences

An agent chooses among prizes (A, B, etc.) and lotteries, i.e., situations with uncertain prizes

Lottery $L = [p, A;\ (1-p), B]$



Notation:

$A \succ B$	A preferred to B
$A \sim B$	indifference between A and B
$A \succsim B$	B not preferred to A

Rational preferences

Idea: preferences of a rational agent must obey constraints.

Rational preferences $\Rightarrow$

behavior describable as maximization of expected utility

Constraints:

Orderability

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

Transitivity

$$(A \succ B) \wedge (B \succ C) \;\Rightarrow\; (A \succ C)$$

Continuity

$$A \succ B \succ C \;\Rightarrow\; \exists\, p \;\; [p, A;\; 1-p, C] \sim B$$

Substitutability

$$A \sim B \;\Rightarrow\; [p, A;\; 1-p, C] \sim [p, B; 1-p, C]$$

Monotonicity

$$A \succ B \;\Rightarrow\; (p \geq q \;\Leftrightarrow\; [p, A;\; 1-p, B] \succsim [q, A;\; 1-q, B])$$

Rational preferences contd.

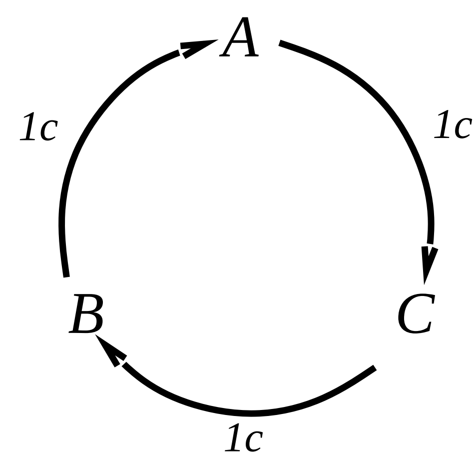
Violating the constraints leads to self-evident irrationality

For example: an agent with intransitive preferences can be induced to give away all its money

If $B \succ C$, then an agent who has C would pay (say) 1 cent to get B

If $A \succ B$, then an agent who has B would pay (say) 1 cent to get A

If $C \succ A$, then an agent who has A would pay (say) 1 cent to get C



Maximizing expected utility

Theorem (Ramsey, 1931; von Neumann and Morgenstern, 1944):
Given preferences satisfying the constraints
there exists a real-valued function U such that

$$U(A) \geq U(B) \quad \Leftrightarrow \quad A \succsim B$$
$$U([p_1, S_1;\ \ldots\ ;\ p_n, S_n]) = \sum_i p_i U(S_i)$$

MEU principle:
Choose the action that maximizes expected utility

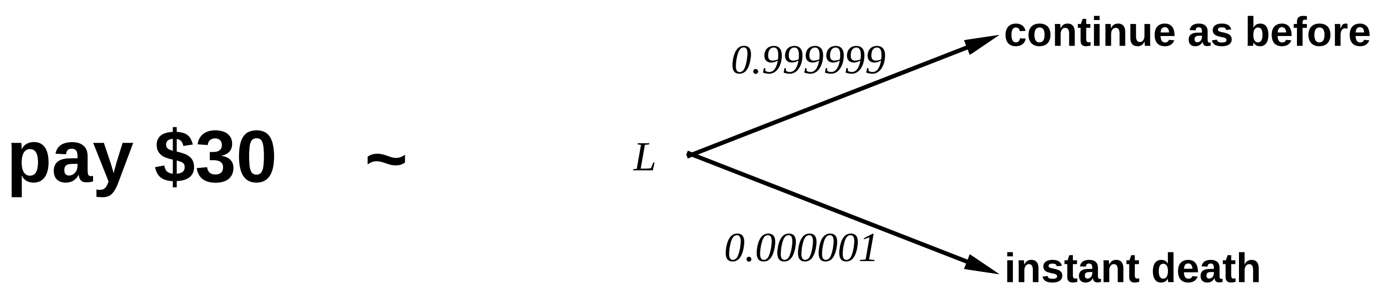
Note: an agent can be entirely rational (consistent with MEU)
without ever representing or manipulating utilities and probabilities

E.g., a lookup table for perfect tictactoe

Utilities

Utilities map states to real numbers. Which numbers?

Standard approach to assessment of human utilities:
- compare a given state A to a standard lottery L_p that has
  - "best possible prize" $u_\top$ with probability p
  - "worst possible catastrophe" $u_\bot$ with probability $(1-p)$
- adjust lottery probability p until $A \sim L_p$



Utility scales

Normalized utilities: $u_\top = 1.0$, $u_\perp = 0.0$

Micromorts: one-millionth chance of death
useful for Russian roulette, paying to reduce product risks, etc.

QALYs: quality-adjusted life years
useful for medical decisions involving substantial risk

Note: behavior is **invariant** w.r.t. $+$ve linear transformation

$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

With deterministic prizes only (no lottery choices), only
ordinal utility can be determined, i.e., total order on prizes

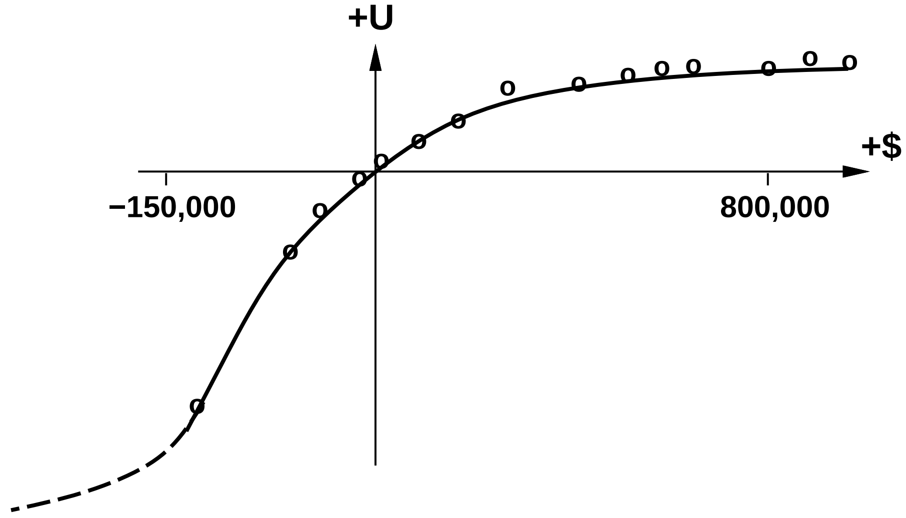
Money

Money does **not** behave as a utility function

Given a lottery L with expected monetary value $EMV(L)$,
usually $U(L) < U(EMV(L))$, i.e., people are risk-averse

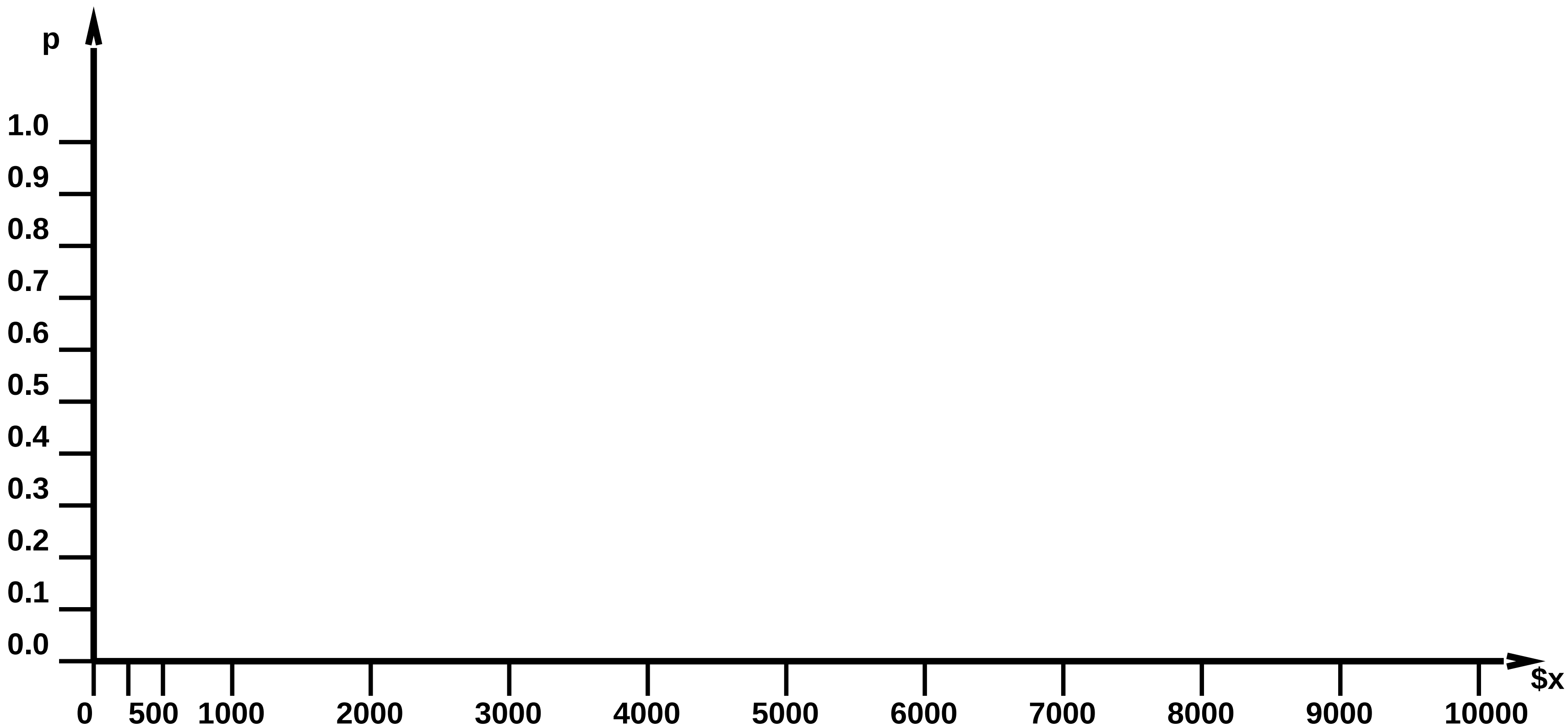
Utility curve: for what probability p am I indifferent between a prize x and a lottery $[p, \$M;\ (1-p), \$0]$ for large M?

Typical empirical data, extrapolated with risk-prone behavior:



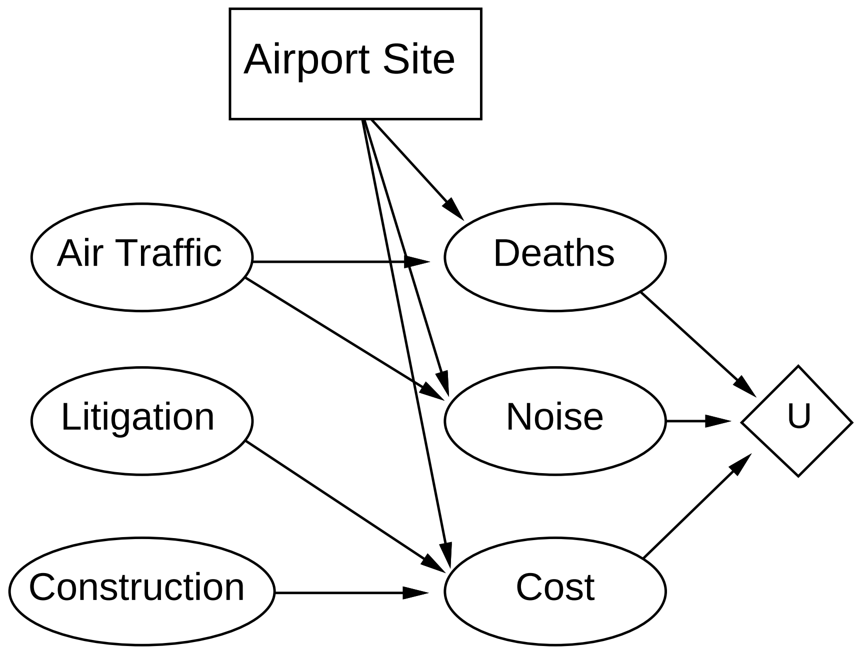
Student group utility

For each x, adjust p until half the class votes for lottery (M=10,000)



Decision networks

Add action nodes and utility nodes to belief networks
to enable rational decision making



Algorithm:

For each value of action node
    compute expected value of utility node given action, evidence
Return MEU action

Multiattribute utility

How can we handle utility functions of many variables $X_1 \ldots X_n$?
E.g., what is $U(Deaths, Noise, Cost)$?

How can complex utility functions be assessed from preference behaviour?

Idea 1: identify conditions under which decisions can be made without complete identification of $U(x_1, \ldots, x_n)$

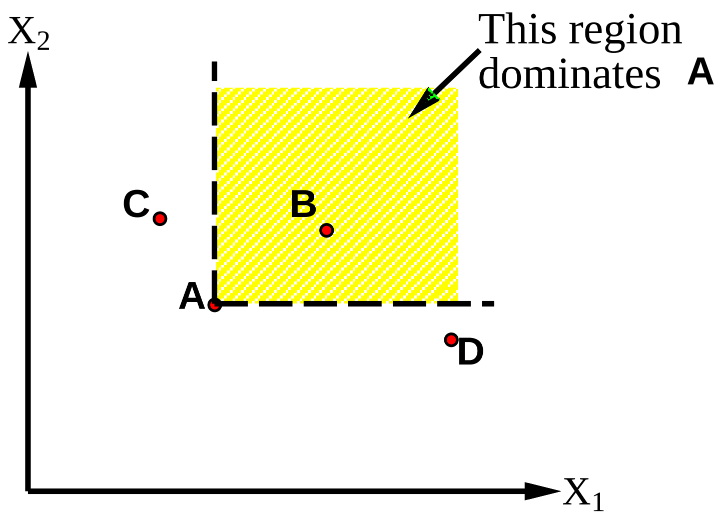
Idea 2: identify various types of **independence** in preferences and derive consequent canonical forms for $U(x_1, \ldots, x_n)$

Strict dominance

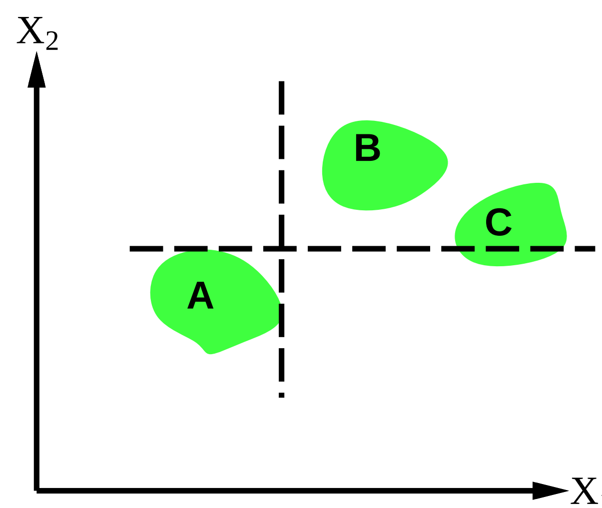
Typically define attributes such that U is monotonic in each

Strict dominance: choice B strictly dominates choice A iff

$$\forall i \;\; X_i(B) \geq X_i(A) \qquad \text{(and hence } U(B) \geq U(A)\text{)}$$



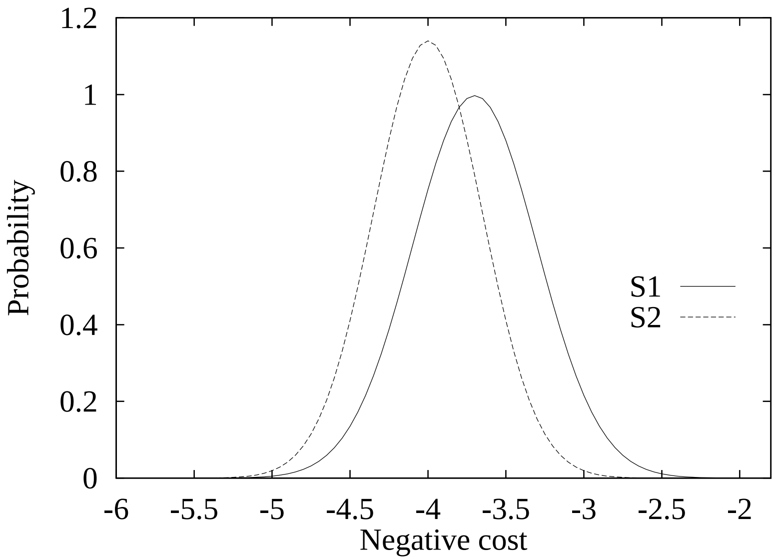
Deterministic attributes

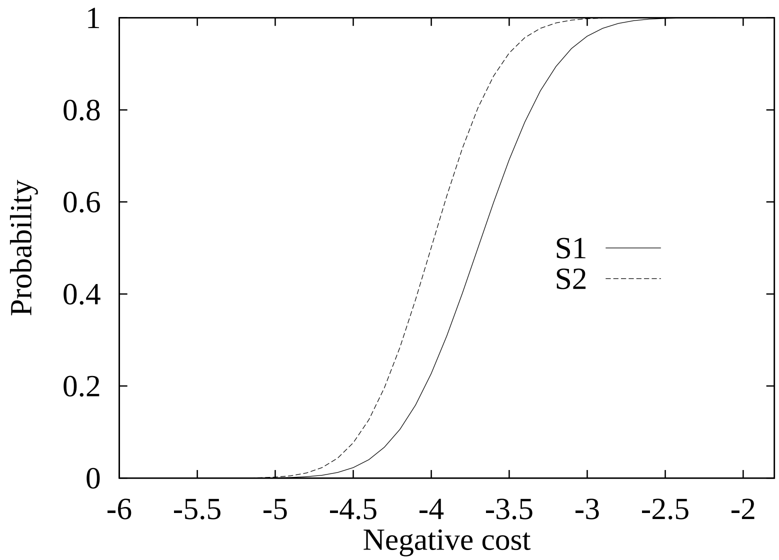


Uncertain attributes

Strict dominance seldom holds in practice

Stochastic dominance





Distribution p_1 stochastically dominates distribution p_2 iff

$$\forall\, t \quad \int_{-\infty}^{t} p_1(x)dx \leq \int_{-\infty}^{t} p_2(t)dt$$

If U is monotonic in x, then A_1 with outcome distribution p_1 stochastically dominates A_2 with outcome distribution p_2:

$$\int_{-\infty}^{\infty} p_1(x)U(x)dx \geq \int_{-\infty}^{\infty} p_2(x)U(x)dx$$

Multiattribute case: stochastic dominance on all attributes $\Rightarrow$ optimal

Stochastic dominance contd.

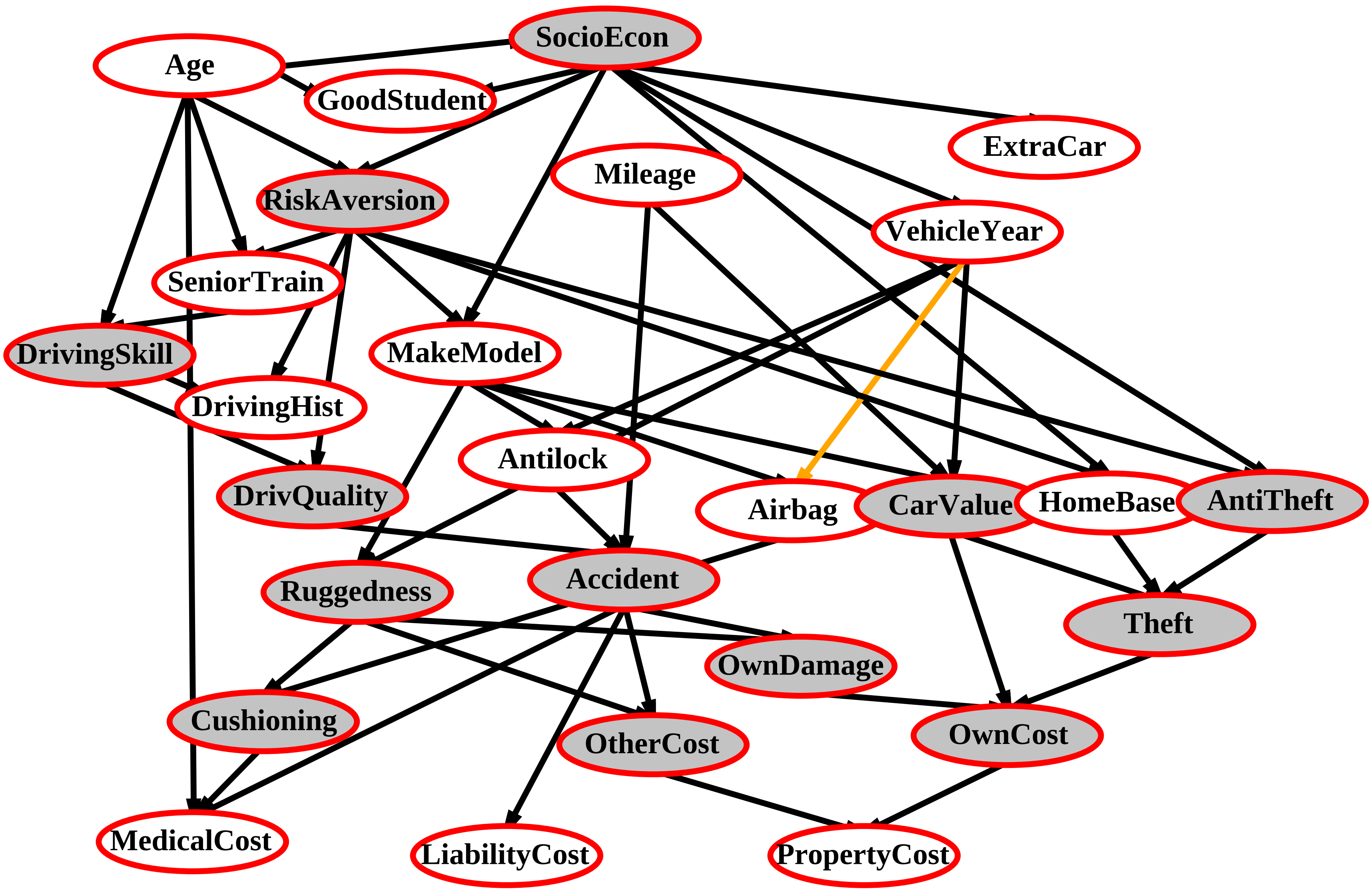
Stochastic dominance can often be determined without exact distributions using **qualitative** reasoning

E.g., construction cost increases with distance from city
S_1 is closer to the city than S_2
$\Rightarrow$ S_1 stochastically dominates S_2 on cost

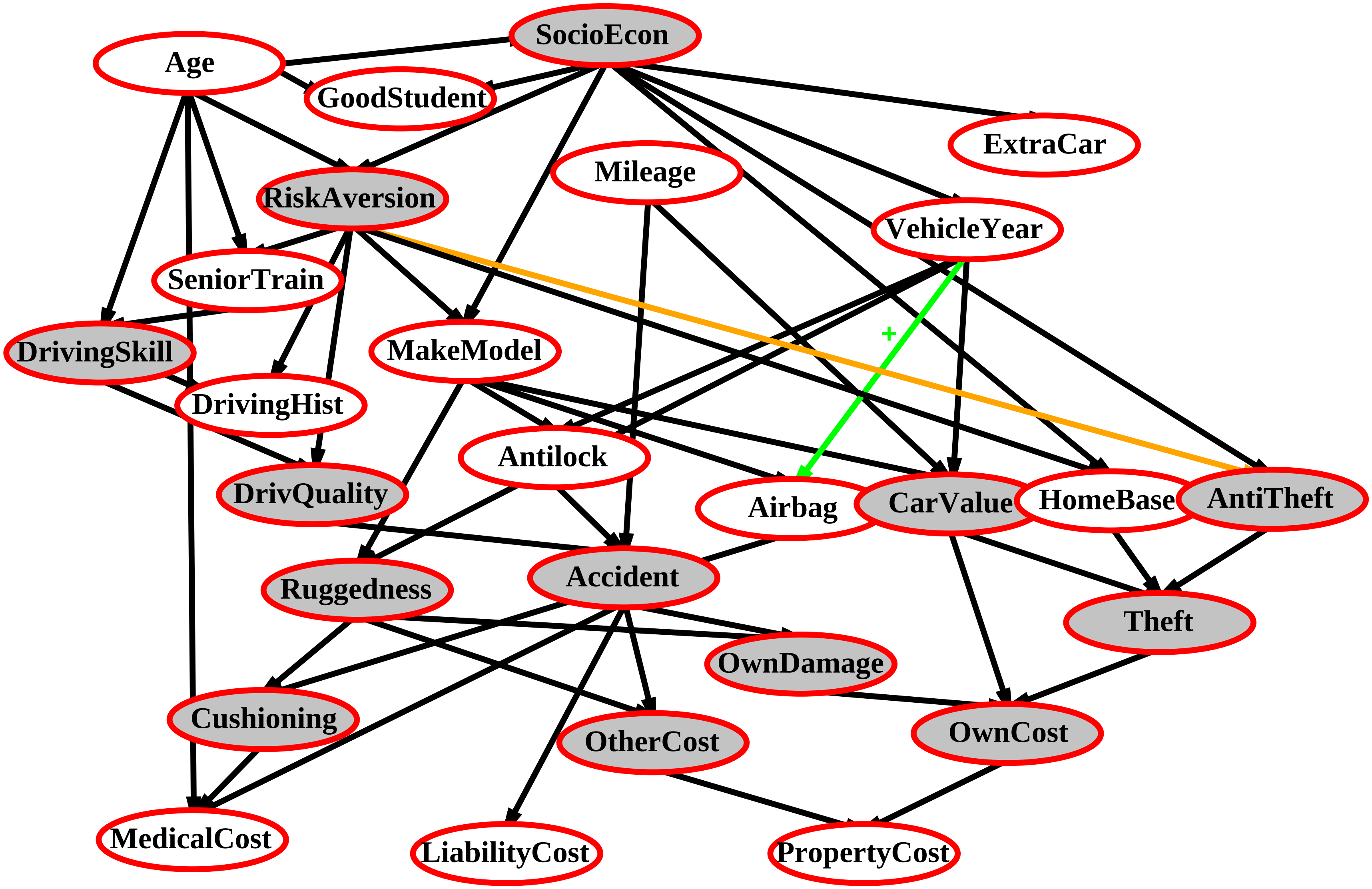
E.g., injury increases with collision speed

Can annotate belief networks with stochastic dominance information:
$X \xrightarrow{+} Y$ (X positively influences Y) means that
For every value $\mathbf{z}$ of Y's other parents $\mathbf{Z}$
$\forall\, x_1, x_2 \;\; x_1 \geq x_2 \Rightarrow \mathbf{P}(Y|x_1, \mathbf{z})$ stochastically dominates $\mathbf{P}(Y|x_2, \mathbf{z})$

Label the arcs + or −



Label the arcs + or −



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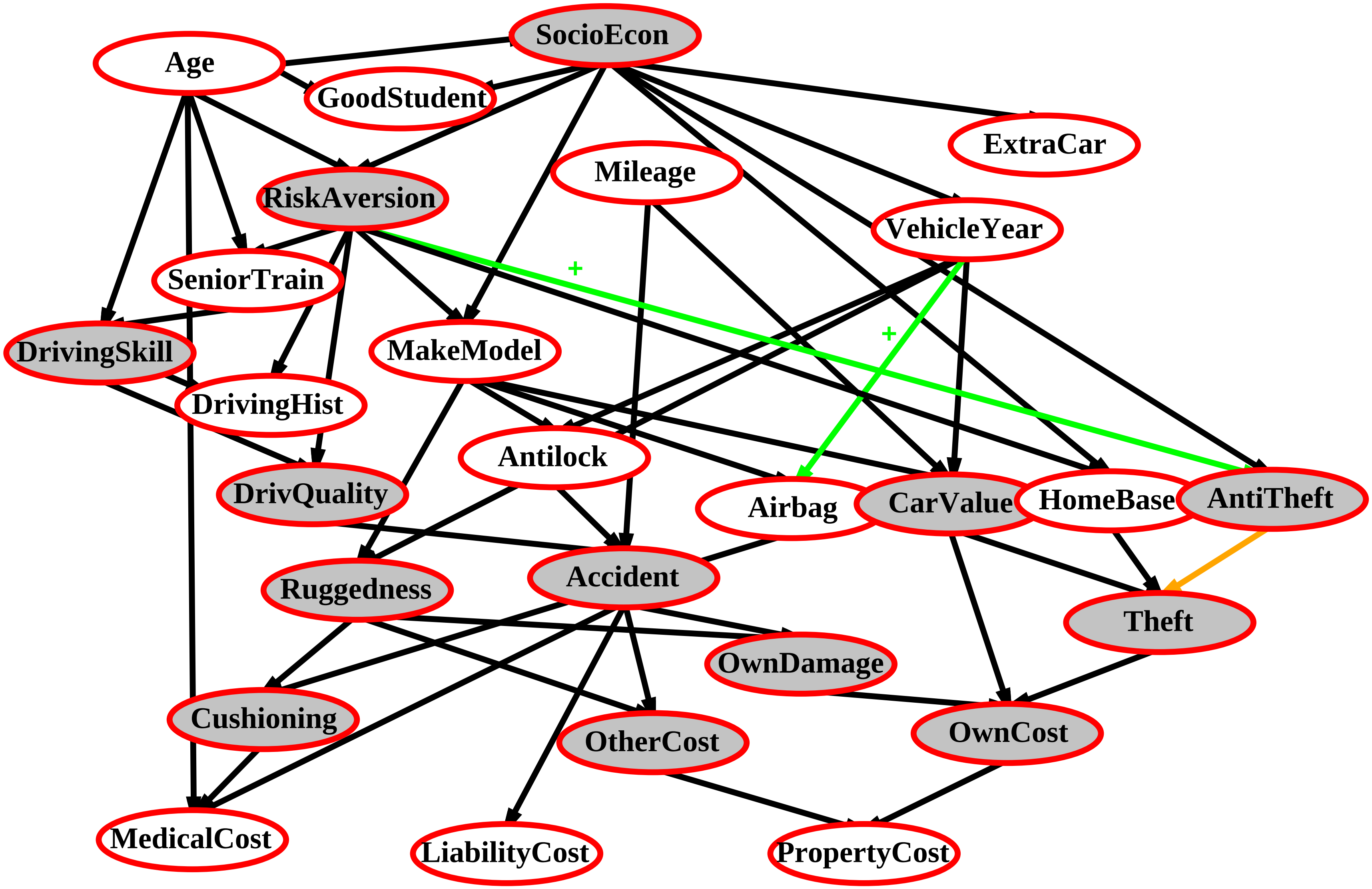

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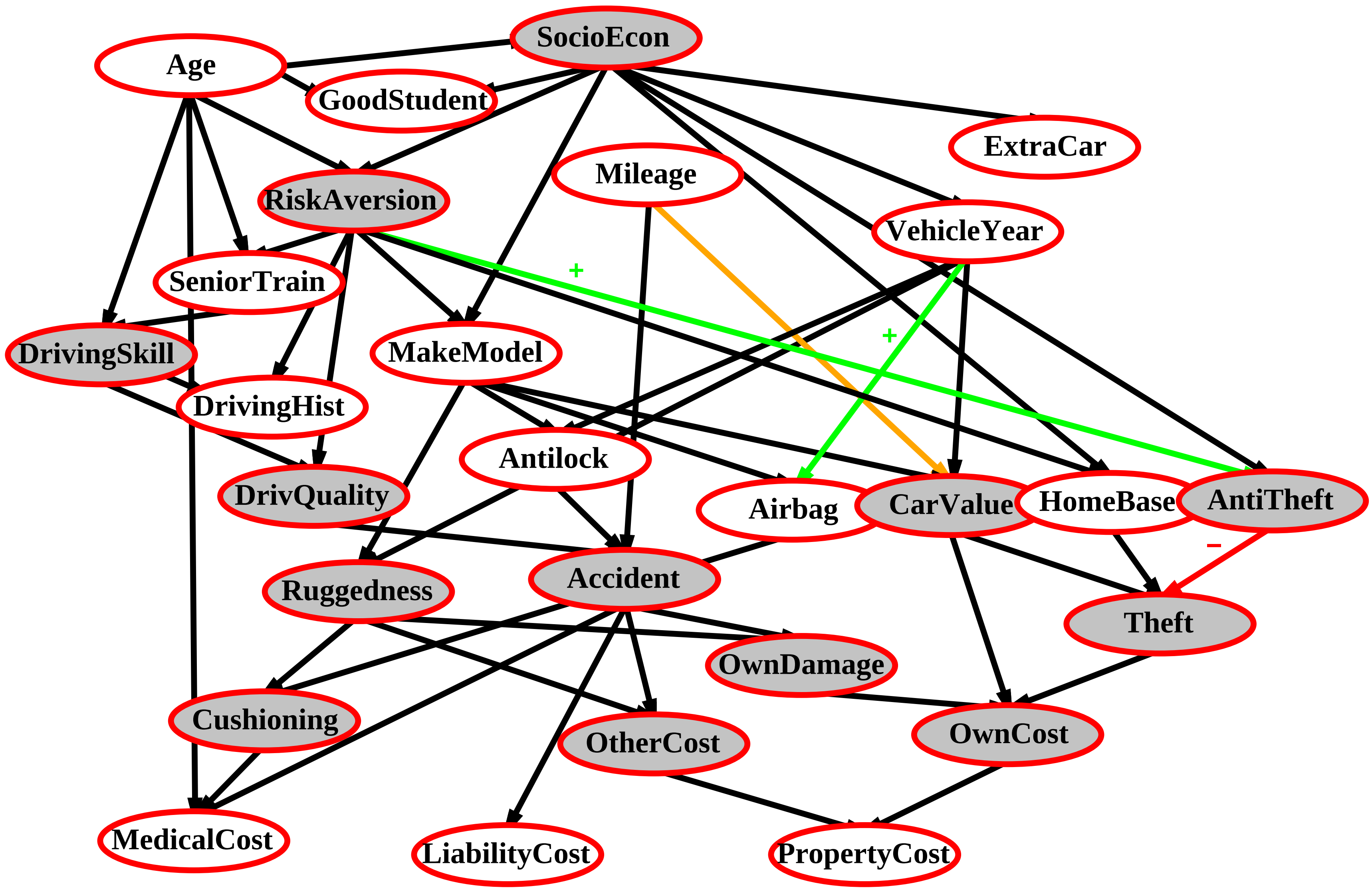

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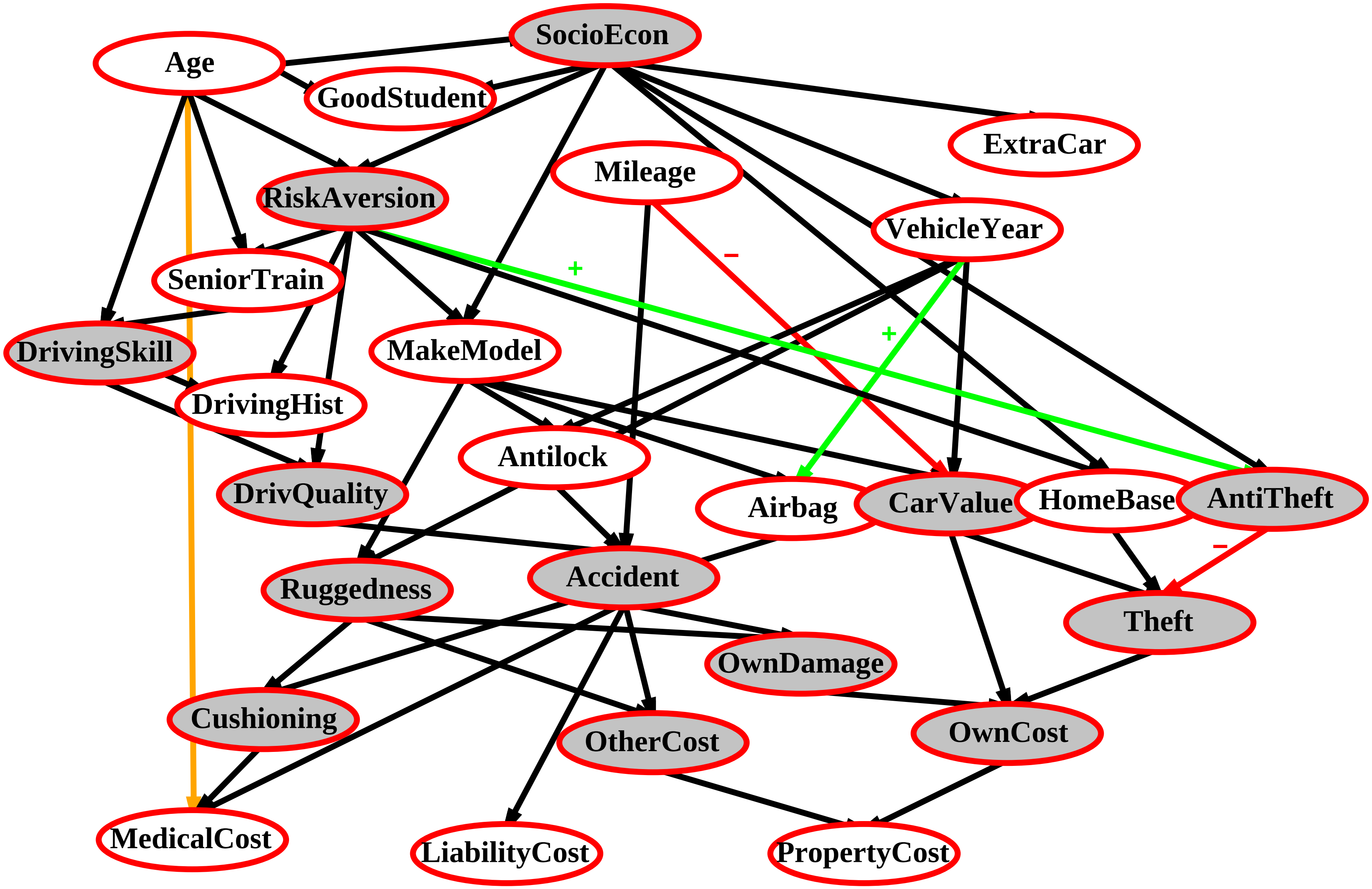

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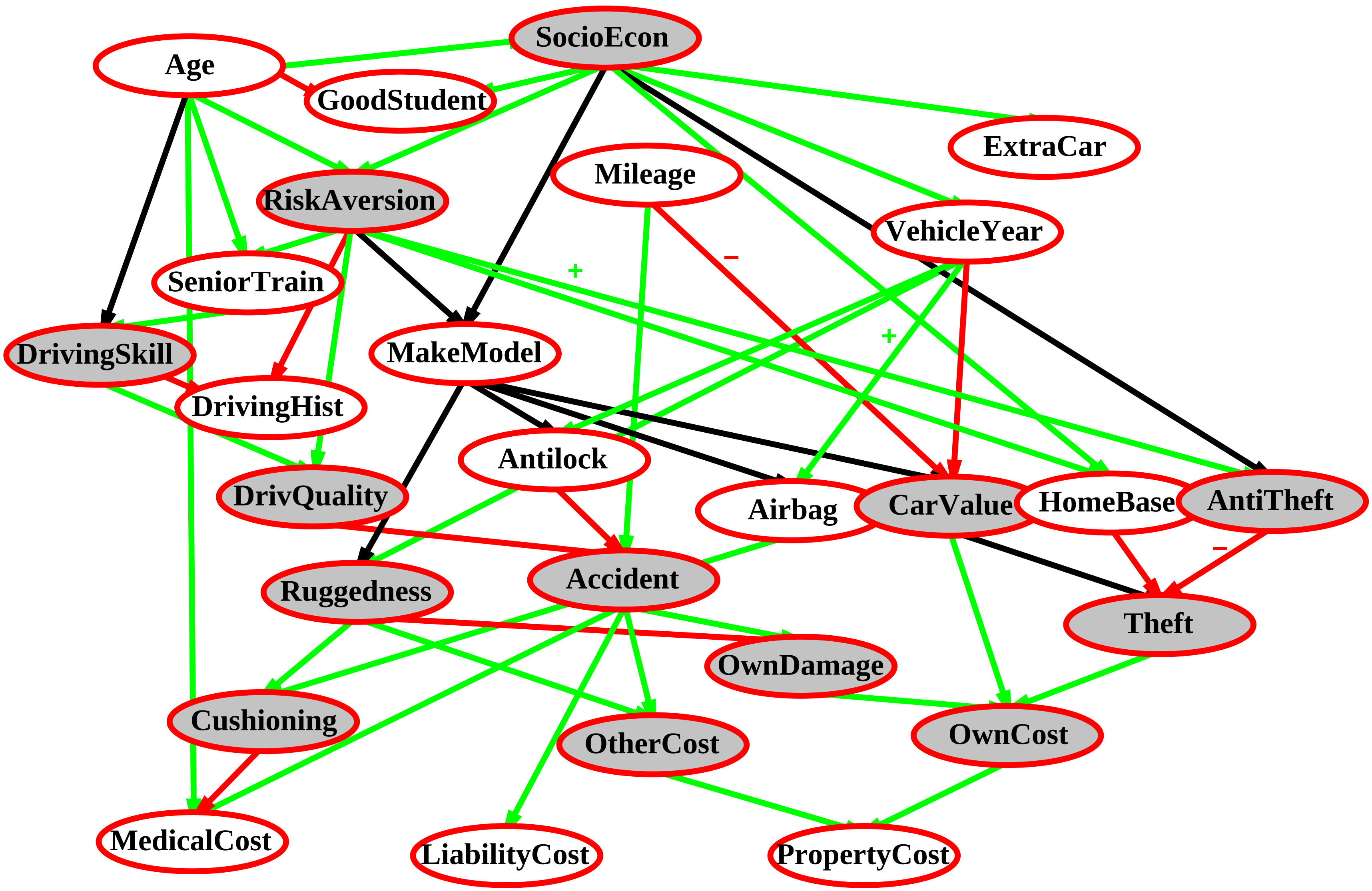

Preference structure: Deterministic

X_1 and X_2 preferentially independent of X_3 iff
preference between $\langle x_1, x_2, x_3 \rangle$ and $\langle x_1', x_2', x_3 \rangle$
does not depend on x_3

E.g., $\langle Noise, Cost, Safety \rangle$:
$\langle$20,000 suffer, \$4.6 billion, 0.06 deaths/mpm$\rangle$ vs.
$\langle$70,000 suffer, \$4.2 billion, 0.06 deaths/mpm$\rangle$

Theorem (Leontief, 1947): if every pair of attributes is P.I. of its complement, then every subset of attributes is P.I of its complement: mutual P.I..

Theorem (Debreu, 1960): mutual P.I. $\Rightarrow$ $\exists$ additive value function:

$$V(S) = \Sigma_i V_i(X_i(S))$$

Hence assess n single-attribute functions; often a good approximation

Preference structure: Stochastic

Need to consider preferences over lotteries:
$\mathbf{X}$ is utility-independent of $\mathbf{Y}$ iff
preferences over lotteries in $\mathbf{X}$ do not depend on $\mathbf{y}$

Mutual U.I.: each subset is U.I of its complement
$\Rightarrow\ \exists$ multiplicative utility function:

$$U = k_1U_1 + k_2U_2 + k_3U_3 + k_1k_2U_1U_2 + k_2k_3U_2U_3 + k_3k_1U_3U_1 + k_1k_2k_3U_1U_2U_3$$

Routine procedures and software packages for generating preference tests to identify various canonical families of utility functions

Value of information

Idea: compute value of acquiring each possible piece of evidence
Can be done **directly from decision network**

Example: buying oil drilling rights
  Two blocks A and B, exactly one has oil, worth k
  Prior probabilities 0.5 each, mutually exclusive
  Current price of each block is $k/2$
  "Consultant" offers accurate survey of A. Fair price?

Solution: compute expected value of information
  $=$ expected value of best action given the information
    minus expected value of best action without information
Survey may say "oil in A" or "no oil in A", **prob. 0.5 each** (given!)
  $=$ [$0.5 \times$ value of "buy A" given "oil in A"
    $+ \; 0.5 \times$ value of "buy B" given "no oil in A"]
    – 0
  $= (0.5 \times k/2) + (0.5 \times k/2) - 0 = k/2$

General formula

Current evidence E, current best action α
Possible action outcomes S_i, potential new evidence E_j

$$EU(\alpha|E) = \max_a \Sigma_i\, U(S_i)\ P(S_i|E, a)$$

Suppose we knew $E_j = e_{jk}$, then we would choose $\alpha_{e_{jk}}$ s.t.

$$EU(\alpha_{e_{jk}}|E, E_j = e_{jk}) = \max_a \Sigma_i\, U(S_i)\ P(S_i|E, a, E_j = e_{jk})$$

E_j is a random variable whose value is *currently* unknown
$\Rightarrow$ must compute expected gain over all possible values:

$$VPI_E(E_j) = \left(\Sigma_k\ P(E_j = e_{jk}|E) EU(\alpha_{e_{jk}}|E, E_j = e_{jk})\right) - EU(\alpha|E)$$

(VPI = value of perfect information)

Properties of VPI

Nonnegative—in **expectation**, not **post hoc**

$$\forall\, j, E \;\; VPI_E(E_j) \geq 0$$

Nonadditive—consider, e.g., obtaining E_j twice

$$VPI_E(E_j, E_k) \neq VPI_E(E_j) + VPI_E(E_k)$$

Order-independent

$$VPI_E(E_j, E_k) = VPI_E(E_j) + VPI_{E,E_j}(E_k) = VPI_E(E_k) + VPI_{E,E_k}(E_j)$$

Note: when more than one piece of evidence can be gathered, maximizing VPI for each to select one is not always optimal
$\Rightarrow$ evidence-gathering becomes a **sequential** decision problem

Qualitative behaviors

a) Choice is obvious, information worth little
b) Choice is nonobvious, information worth a lot
c) Choice is nonobvious, information worth little