LOCAL SEARCH ALGORITHMS

Chapter 4, Sections 3–4

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Outline

- \diamond Hill-climbing
- \diamondsuit Simulated annealing
- ♦ Genetic algorithms (briefly)
- \diamond Local search in continuous spaces (very briefly)

Iterative improvement algorithms

In many optimization problems, **path** is irrelevant; the goal state itself is the solution

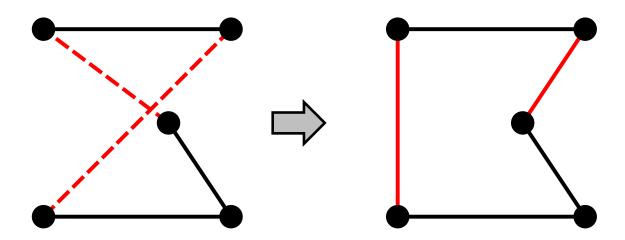
Then state space = set of "complete" configurations; find **optimal** configuration, e.g., TSP or, find configuration satisfying constraints, e.g., timetable

In such cases, can use iterative improvement algorithms; keep a single "current" state, try to improve it

Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

Start with any complete tour, perform pairwise exchanges

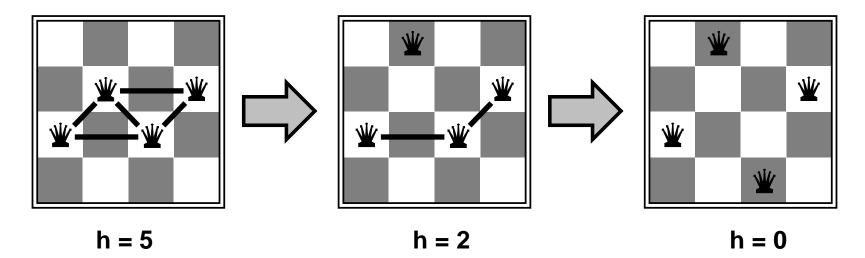


Variants of this approach get within 1% of optimal very quickly with thousands of cities

Example: *n*-queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

Move a queen to reduce number of conflicts



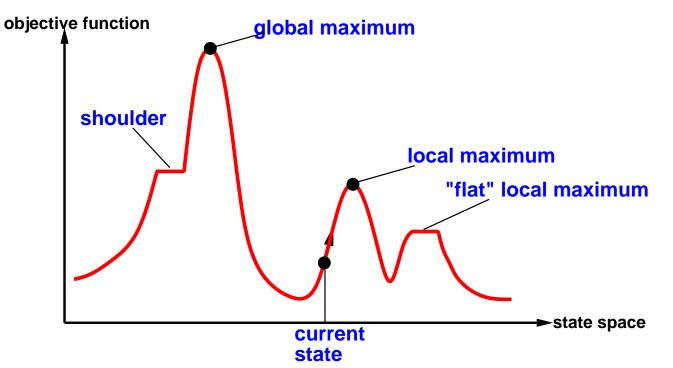
Almost always solves *n*-queens problems almost instantaneously for very large *n*, e.g., n = 1million

Hill-climbing (or gradient ascent/descent)

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"Like climbing Everest in thick fog with amnesia"
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Hill-climbing contd.

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete Random sideways moves ©escape from shoulders ©loop on flat maxima

Simulated annealing

Idea: escape local maxima by allowing some "bad" moves but gradually decrease their size and frequency

```
function SIMULATED-ANNEALING (problem, schedule) returns a solution state
inputs: problem, a problem
           schedule, a mapping from time to "temperature"
local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps
current \leftarrow Make-Node(INITIAL-STATE[problem])
for t \leftarrow 1 to \infty do
     T \leftarrow schedule[t]
     if T = 0 then return current
     next \leftarrow a randomly selected successor of current
     \Delta E \leftarrow \text{VALUE}[next] - \text{VALUE}[current]
     if \Delta E > 0 then current \leftarrow next
     else current \leftarrow next only with probability e^{\Delta E/T}
```

Properties of simulated annealing

At fixed "temperature" $T,\ {\rm state}\ {\rm occupation}\ {\rm probability}\ {\rm reaches}\ {\rm Boltzman}\ {\rm distribution}$

 $p(x) = \alpha e^{\frac{E(x)}{kT}}$

T decreased slowly enough \Longrightarrow always reach best state x^* because $e^{\frac{E(x^*)}{kT}}/e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*)-E(x)}{kT}} \gg 1$ for small T

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.

Local beam search

Idea: keep k states instead of 1; choose top k of all their successors

Not the same as k searches run in parallel! Searches that find good states recruit other searches to join them

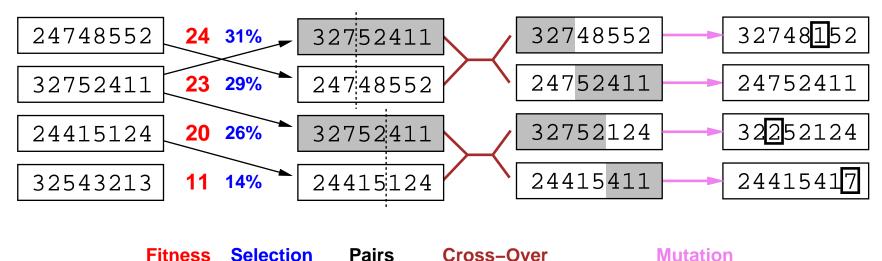
Problem: quite often, all k states end up on same local hill

ldea: choose k successors randomly, biased towards good ones

Observe the close analogy to natural selection!

Genetic algorithms

= stochastic local beam search + generate successors from **pairs** of states



Fitness Selection

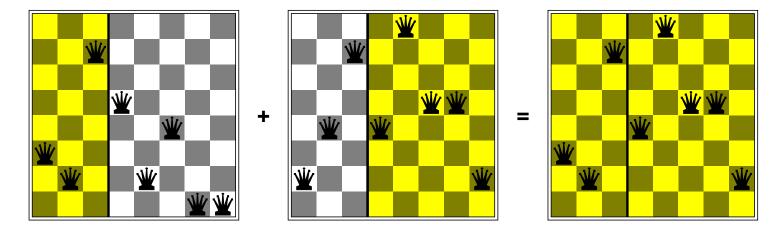
Pairs

Mutation

Genetic algorithms contd.

GAs require states encoded as strings (GPs use programs)

Crossover helps iff substrings are meaningful components



 $GAs \neq evolution: e.g., real genes encode replication machinery!$

Continuous state spaces

Suppose we want to site three airports in Romania:

- 6-D state space defined by (x_1,y_2) , (x_2,y_2) , (x_3,y_3)
- objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$ sum of squared distances from each city to nearest airport

Discretization methods turn continuous space into discrete space, e.g., empirical gradient considers $\pm \delta$ change in each coordinate

Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce f , e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one city). Newton-Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x})\nabla f(\mathbf{x})$ to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$